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Approximate Analysis of Axisymmetric Supersonic Base Flows with Injection

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Abstract

A SIMPLE analysis of the base pressure on a projectile with mass addition was sought, so that base drag reduction due to injection could be estimated. A numerical treatment was rejected, since only limited success has been achieved with this approach for simpler problems of the same type.[1] The Crocco-Lees[2,3,4] approach was not used, since it does not permit easy extension for injection[3] or axisymmetric geometry.[5,6] The approach of Refs. 7 and 8 that was successful for planar problems with injection was chosen as the starting point. That approach involves separate models for separate regions—the main mixing region, the corner/base region, and a recirculation region for low injection rates joined together.

The main problem in any axisymmetric analysis is the selection of a pressure-angle relationship. For the planar arrangement, we used[9]

$$C_p = 2\Delta\theta/\sqrt{M^2-1} \qquad (1)$$

The axisymmetric geometry is complicated, since the distance from the axis enters the pressure-angle relation. All analyses of the type of Refs. 2-8 are best formulated with initial conditions at the "viscous throat"; one then integrates upstream to the base. The difficulty is that the size (radius) of the mixing zone at the viscous throat in relation to the base size (radius) is unknown until the computation is complete. One could start at the base and integrate down through the viscous throat, but numerical passage through the associated singularity is difficult.[5,6,10] Here, we have developed an approximate, heuristic form of Eq. (1) for axisymmetric base flows.

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From the viscous throat upstream to the boundary between the main mixing region and the near-base region, the inner flow is taken as one dimensional with mass entrainment from the outer stream. The equations in Ref. 11 are used

$$\frac{dM^2}{M^2} = -2\frac{[1+(\gamma-1/2)M^2]}{(1-M^2)}\frac{dA}{A} - \frac{(1+\gamma M^2)}{2}\frac{dT_0}{T_0}$$
$$+\gamma M^2\frac{U_e}{V}\frac{d\dot{m}}{\dot{m}} - (1+\gamma M^2)\frac{d\dot{m}}{\dot{m}} \qquad (2)$$

We must model turbulent transport which appears as entrainment, i.e., $d\dot{m}/dx$. Crocco and Lees[2] used, for boundary layer-type flows,

$$d\dot{m}/dx = 0.03\rho_I u_I \qquad (3)$$

Here, it is necessary to account for small entrainment when the mass flux in the freestream equals that in the inner zone. Reference 8 took for the planar case

$$d\dot{m}/dx = K\rho_I u_I\,|1-(\rho u/\rho_I u_I)| \qquad (4)$$

with $K=0.01$. For axisymmetric problems, a straightforward extension is

$$d\dot{m}/dx = 2\pi r K\rho_I u_I\,|1-(\rho u/\rho_I u_I)| \qquad (5)$$

The same value of $K=0.01$ was proven satisfactory by comparison with experiments.

We must now confront the problem of an approximate, axisymmetric pressure-angle relation. We will be willing to accept some crudity to retain the formulation of integrating upstream from the viscous throat. The test of the utility of any approximation will be comparison with relevant experiment. We rejected other approximations, e.g., Ref. 12, since they also did not permit upstream marching.

Consider a uniform, axisymmetric flow that encounters a small angular direction change $\Delta\theta$ and follow a characteristic from a point in the uniform stream down to a streamline which has undergone the $\Delta\theta$ direction change. Starting with the exact relation along such a characteristic[9], one can derive

$$\Delta\nu \equiv \frac{\sqrt{M^2-1}\Delta w}{w} \approx -\Delta\theta\left(1-\frac{\Delta r}{2r}\right) \qquad (6)$$

Then, we can say

$$C_p \approx \frac{-2\Delta w}{w} \approx \frac{2\Delta\theta}{\sqrt{M^2-1}}\left(1-\frac{\Delta r}{2r}\right) \qquad (1a)$$

Our task is now to derive a simple approximation for the term in parentheses, and we have considered for that purpose the solution for a line of sources in a uniform stream.[9] If the strength is $f(\xi)=a\xi$, the velocity perturbation components become[9]

$$u(x,r) = -a\cosh^{-1}\left(\frac{x}{\lambda r}\right) \qquad v(x,r) = a\lambda\sqrt{\left(\frac{x}{\lambda r}\right)^2-1} \qquad (7)$$

where $\lambda \equiv \sqrt{M^2-1}$. x is measured from $\xi=0$. The contribution of $f(\xi)$ to the flow at (x,r) is only from points where $\xi \le (x-\lambda r)$. Assuming that the range of important points, $0\le\xi\le\ell$, can be approximated by $\ell=\lambda r$, we get

$$u = -a\cosh^{-1}(2) = -1.32a, \quad v = a\lambda\sqrt{3} \qquad (8)$$

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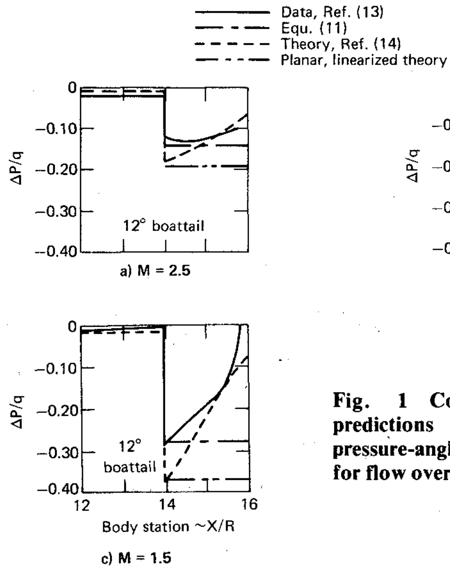

Fig. 1 Comparison of the predictions of the approximate pressure-angle relation and data for flow over boattailed bodies.

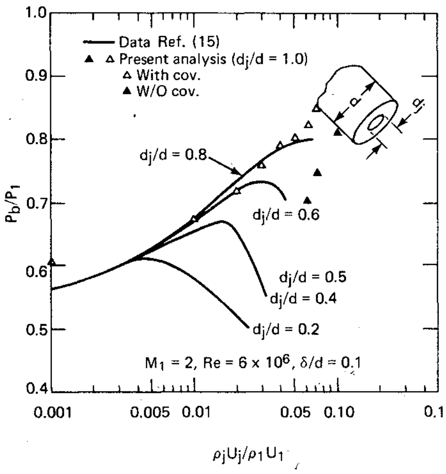

Fig. 2 Comparison of predictions and data for base pressure at $M_l = 2.0$.

and

$$\theta \approx v(x,r)/U = a\lambda\sqrt{3}/U \tag{9a}$$

which gives

$$a = U\theta/\lambda\sqrt{3} \tag{9b}$$

Taking the pressure coefficient as

$$C_p \approx -2u/U \tag{10}$$

the result is

$$C_p = (1.32/\sqrt{3})\,(2\Delta\theta/\sqrt{M^2-1}) \tag{11}$$

Except for the numerical factor, Eq. (11) is identical to Eq. (1).

Is Eq. (11) a reasonable approximation? Our answer will be based upon comparison of predictions with experimental pressures on boattailed bodies. We used the severe case of 12 deg boattails from Ref. 13. The comparisons are shown on Fig. 1 along with the results of the elaborate, axisymmetric theory of Tsien and the planar linearized theory. Clearly, the crudely derived result in Eq. (11) serves reasonably well.

The corner region of the flow is important, since it governs the initial conditions for the shear layer. A new method was developed in Ref. 8. It is presumed that some portion Δ of the boundary layer thickness δ participates in the formation of the shear layer. No assumption as to the value of the Mach number at Δ is introduced. The correct solution was selected by matching with the solution from the main mixing region. One additional refinement was a crude attempt to account for the distorted profile shapes that occur for low base injection cases by introducing estimates of the "covariance" terms involved.

We tested the utility of this analysis by comparison with the tests of Ref. 15, see Fig. 2. Note that the injection region did not cover the entire base as assumed in the analytical model. If adequate, our analysis should compare most favorably with the data for the highest value of d_j/d, viz. 0.8. For $\rho_j u_j/\rho_l u_l < 0.08$, the inclusion of the covariance effects is clearly important. Also, the particular covariance values used are best suited to $0 \leq \rho_j u_j/\rho_l u_l \leq 0.06$, which covers the range of most practical interest. The resulting solution is in good agreement with the data over the whole range of the injection parameter. Other comparisons with data are given in the back-up document.

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